به نام خدا

سیگنال‌ها و سیستم‌ها

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دانشکده فنی دانشگاه گیلان

SIGNALS
&
SYSTEMS

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Lecture-6

سیستم‌های خطیِ تغییرناپذیر با زمان

LTI Systems

- در ادامه، ابتدا به استخراج رابطه‌ی ورودی-خروجی برای سیستم‌های LTI زمان-پیوسته می‌پردازیم.

روند کلی، مانند حالت زمان-گسسته است؛ با این تفاوت که، به دلیل خاص بودنِ ضربه‌ی واحدِ زمانَ-پیوسته (یا همان دلتای دیراک)، از فرایندِ نمایشِ تقریبیِ سیگنال $x(t)$ و سَپس حدگیریَ استفاده می‌شود.

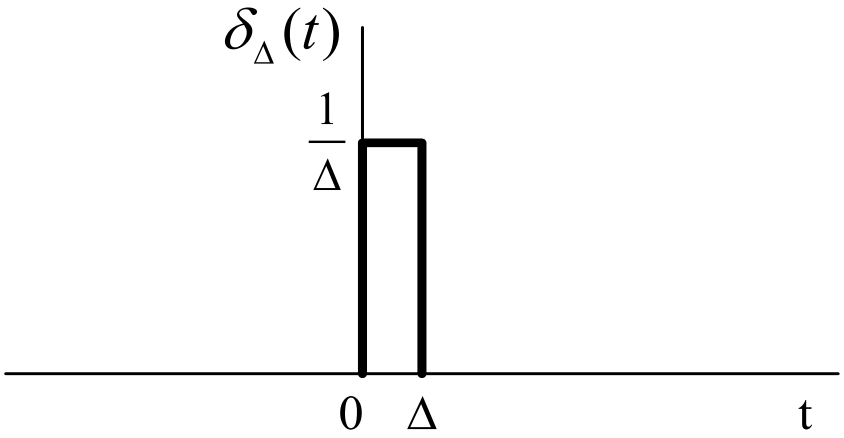
نتیجه‌ی نهایی، نمایشِ انتگرالِ کانولوشن برای سیستم‌های LTI زمان-پیوسته است که به‌صورت زیر بیان می‌گردد :

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau$$

Convolution Integral

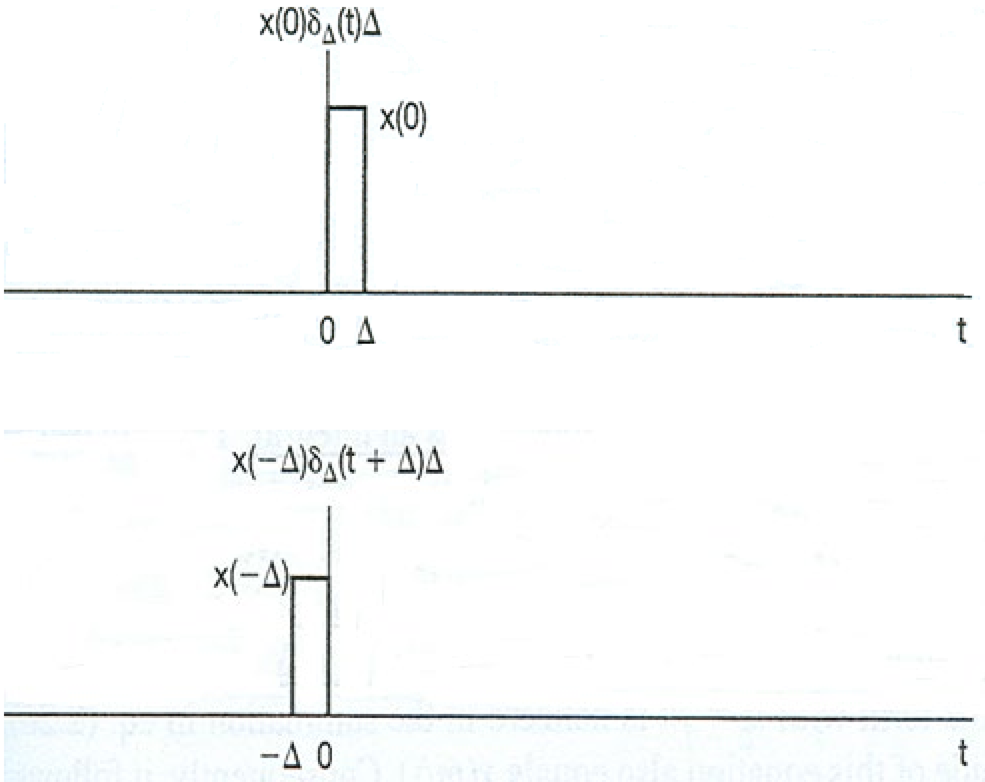
یادآوری

$$\delta_\Delta(t) \triangleq \begin{cases} \dfrac{1}{\Delta}, & 0 \le t < \Delta \\ 0, & otherwise. \end{cases}$$

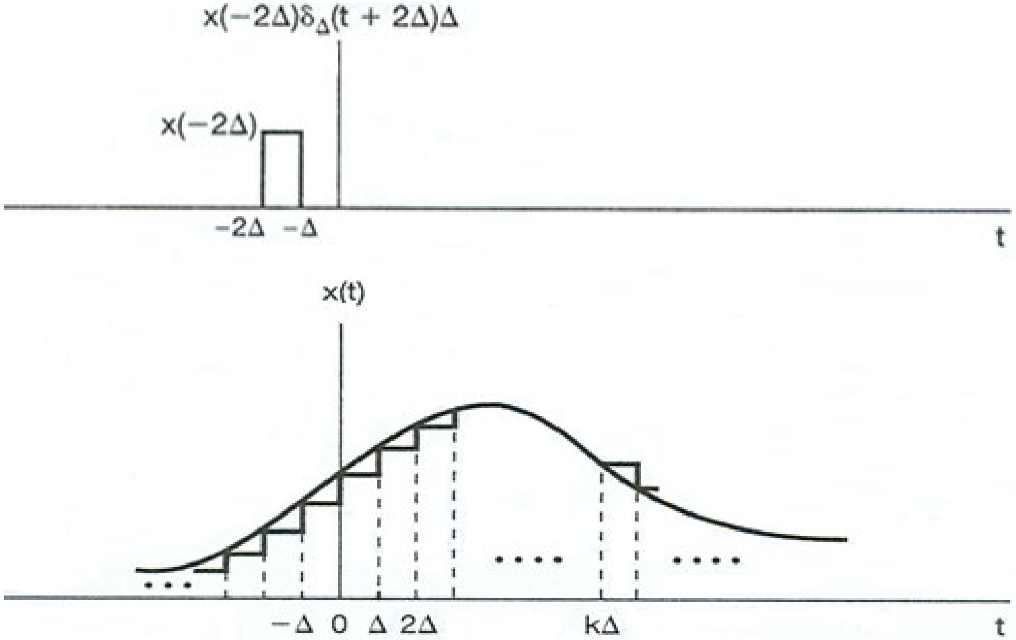


$$\rightarrow \delta_\Delta(t).\Delta \ has \ unit \ amplitude$$

نمایشِ تقریبیِ سیگنال $x(t)$ بر حسب ترکیبِ خطیِ $\delta_\Delta(t)$ و اَنتقال یافته‌هاش



نمایشِ تقریبیِ سیگنال $x(t)$ بر حسب ترکیبِ خطیِ $\delta_{\Delta}(t)$ و اَنتقال یافته‌هاش



$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_{\Delta}(t - k\Delta).\Delta$$

نمایشِ تقریبیِ سیگنال $x(t)$ بر حسب ترکیبِ خطیِ $\delta_\Delta(t)$ و اَنتقال یافته‌هاش

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta).\Delta$$

با میل دادن Δ به سمت صفر ، عبارت فوق به انتگرال زیر بدل می‌شود که نمایشِ دقیق برای $x(t)$ است:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\,\tau$$

$$\Delta \to 0 \quad \Rightarrow \begin{cases} \hat{x} \to x \\ \sum \to \int \\ \delta_\Delta(t-k\Delta) \to \delta(t-\tau) \\ \Delta \to d\tau \end{cases}$$

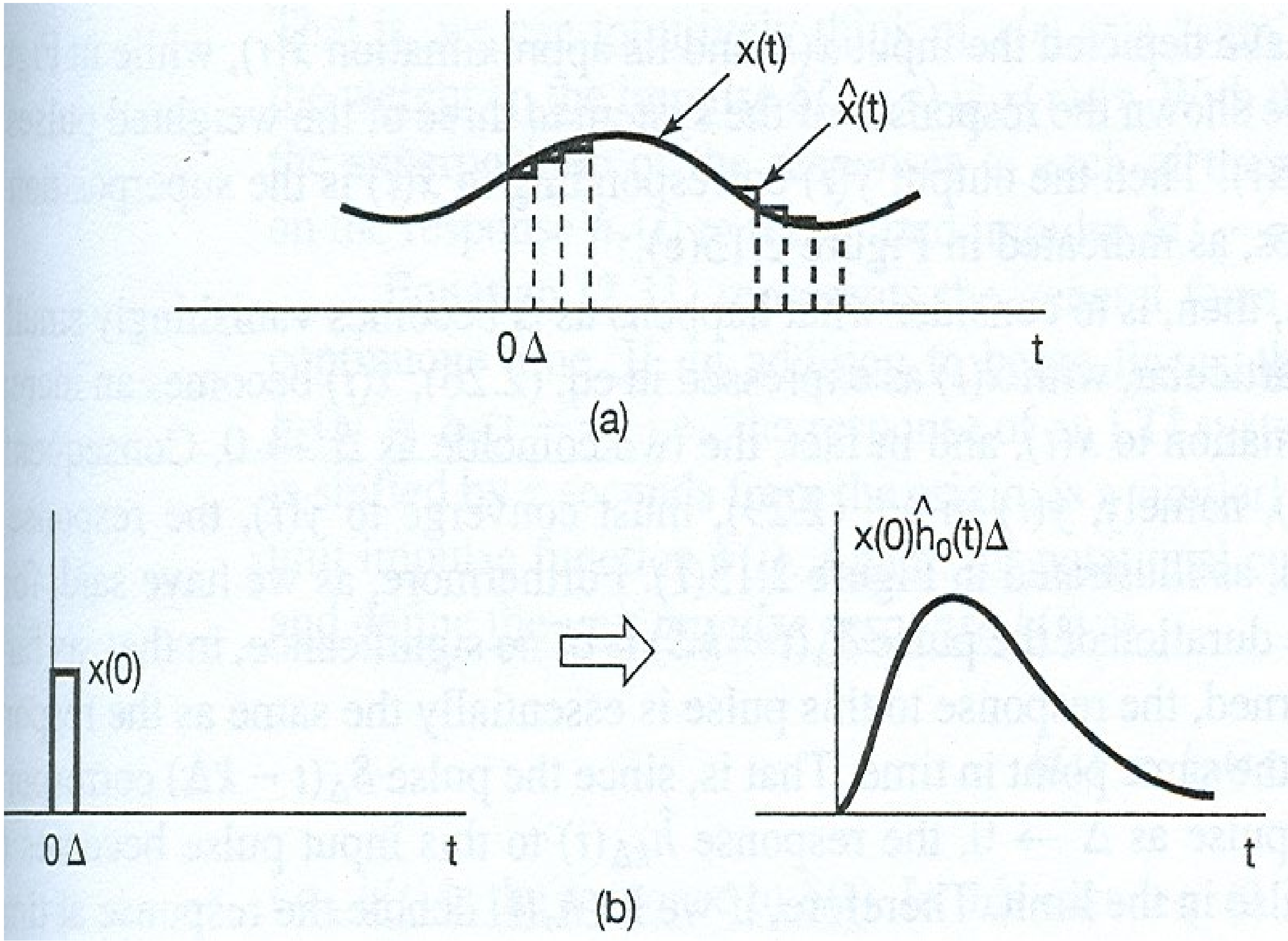
پاسخ سیستمِ خطی به ورودیِ دلخواه $x(t)$

خروجی	سیستم خطی	ورودی
$h_0(t) = h(t)$		$\delta(t)$
$h_\tau(t)$		$\delta(t-\tau)$
$\hat{h}_0(t) = \hat{h}(t)$		$\delta_\Delta(t)$
$\hat{h}_{k\Delta}(t)$		$\delta_\Delta(t-k\Delta)$

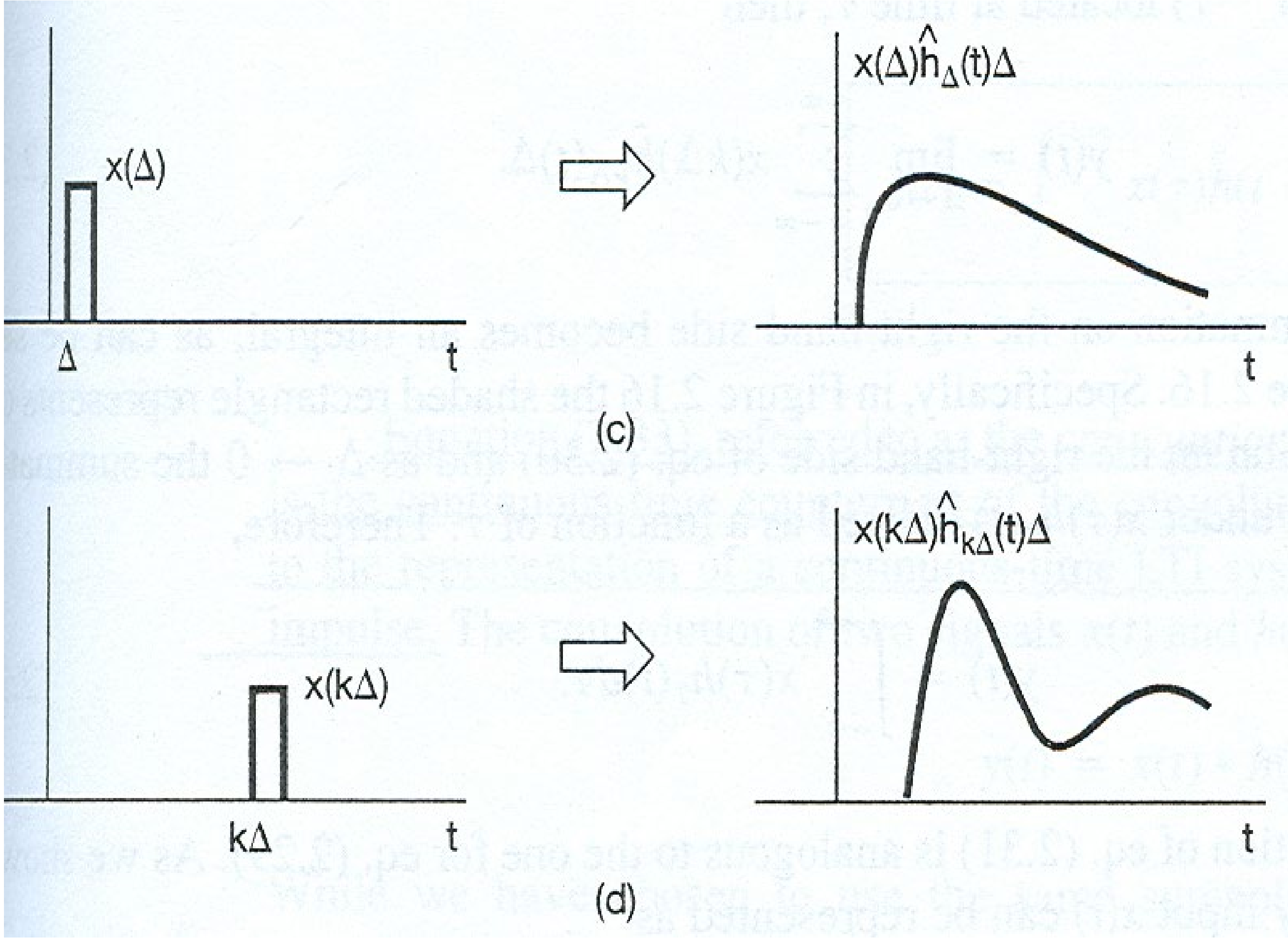
$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta).\Delta$$

$$\hat{y}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\hat{h}_{k\Delta}(t).\Delta$$

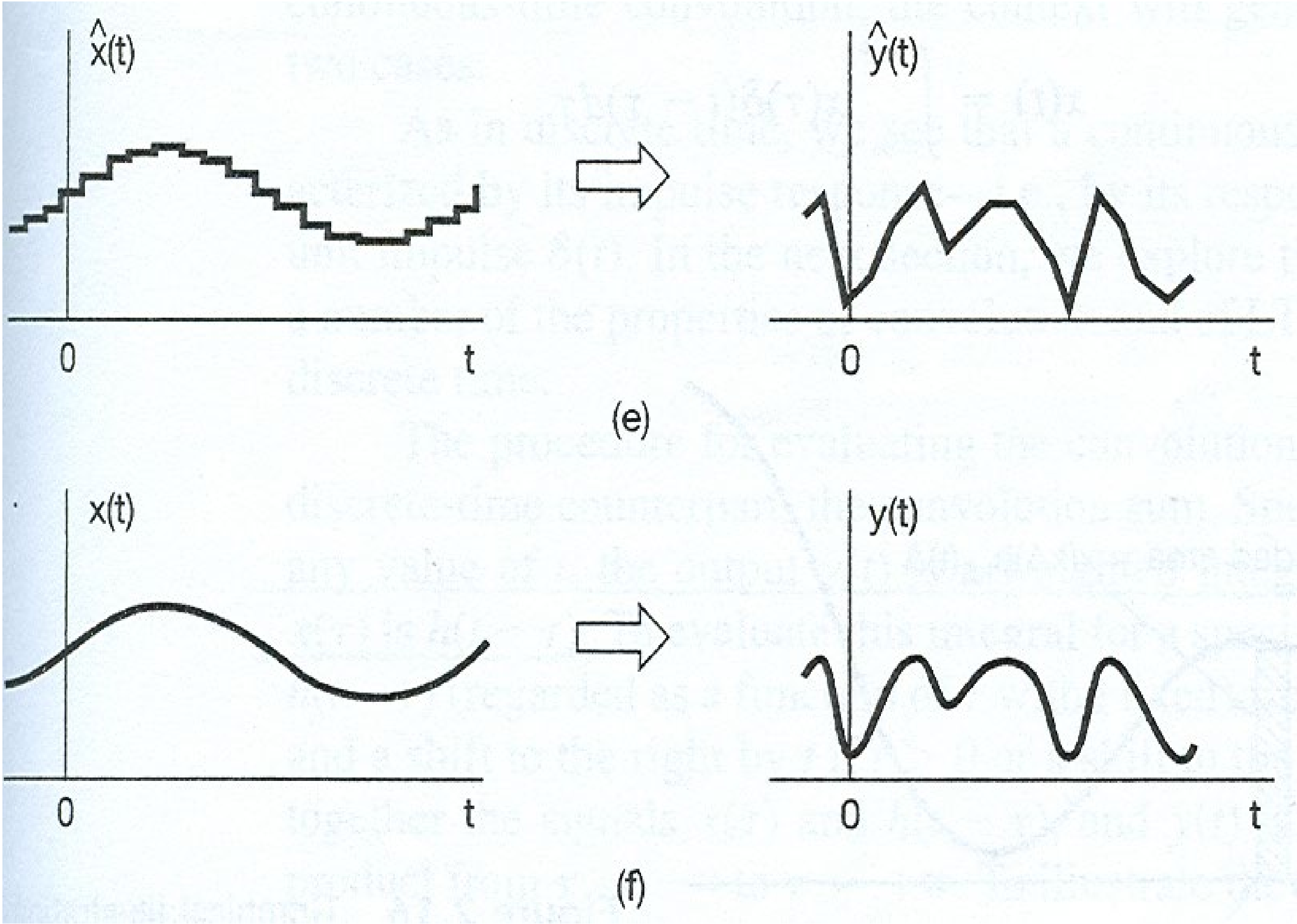
پاسخ سیستمِ خطی به ورودیِ دلخواه $x(t)$



پاسخ سیستمِ خطی به ورودیِ دلخواه $x(t)$



پاسخ سیستمِ خطی به ورودیِ دلخواه $x(t)$



پاسخ سیستمِ خطی به ورودیِ دلخواه $x(t)$

ورودی → سیستم خطی → خروجی

$$\hat{x}(t)=\sum_{k=-\infty}^{\infty}x(k\Delta)\delta_{\Delta}(t-k\Delta).\Delta$$

$\Delta\rightarrow 0$

$$x(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)\,d\tau$$

$$\hat{y}(t)=\sum_{k=-\infty}^{\infty}x(k\Delta)\hat{h}_{k\Delta}(t).\Delta$$

$\Delta\rightarrow 0$

$$y(t)=\int_{-\infty}^{\infty}x(\tau)h_{\tau}(t)\,d\tau$$

$$\boldsymbol{y(t)=\int_{-\infty}^{\infty}x(\tau)h_{\tau}(t)\,d\tau}$$

❖ که $h_{\tau}(t)$ پاسخِ سیستمِ خطی به ورودیِ $\delta(t-\tau)$ است.

پاسخ سیستمِ خطیِ تغییرناپذیر با زمان (LTI) به ورودیِ دلخواه $x(t)$

ورودی → سیستم LTI → خروجی

ورودی	خروجی
$\delta(t)$	$h_0(t) = h(t)$
$\delta(t-\tau)$	$h_\tau(t) = h(t-\tau)$
$\delta_\Delta(t)$	$\hat{h}_0(t) = \hat{h}(t)$
$\delta_\Delta(t-k\Delta)$	$\hat{h}_{k\Delta}(t) = \hat{h}(t-k\Delta)$

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta).\Delta$$

$$\hat{y}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\hat{h}(t-k\Delta).\Delta$$

پاسخ سیستمِ خطیِ تغییرناپذیر با زمان (LTI) به ورودیِ دلخواه $x(t)$

ورودی → سیستم LTI → خروجی

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_{\Delta}(t-k\Delta).\Delta$$

$\Delta \to 0$:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\, d\,\tau$$

$$\hat{y}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\hat{h}(t-k\Delta).\Delta$$

$\Delta \to 0$:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau$$

$$\boldsymbol{y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau}$$

❖ که $h(t)$ پاسخِ سیستمِ LTI به ورودیِ $\delta(t)$ است.

یادآوری

❑ تنها سیستم LTI بی حافظه :

$$h(t) = A\delta(t) \leftarrow y(t) = Ax(t)$$

❑ سیستم LTI علّی :

$$h(t) = 0 \; for \; t < 0$$

❑ سیستم LTI پایدار :

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

سیستم LTI علّی:

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau = \int_{0}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

سیگنال ورودی نیز سیگنالی علّی:

$$= \int_{0}^{t} h(\tau)x(t-\tau)\,d\tau$$

❖ some notes about the limits of the convolution integral in the circuit analysis course

$$= \int_{0}^{t} x(\tau)h(t-\tau)\,d\tau$$

سیگنال ورودی نیز سیگنالی علّی:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{t} x(\tau)h(t-\tau)\,d\tau$$

سیستم LTI علّی:

نحوه‌ی محاسبه‌ی **انتگرال کانولوشن به روش ترسیمی** برای x و h داده شده

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

۱) $\tau \leftarrow t$: $x(\tau) \leftarrow x(t) \;,\; h(\tau) \leftarrow h(t)$

۲) Time-Reversal : $x(-\tau) \leftarrow x(\tau)$

۳) Time-Shift : $x(t-\tau) \leftarrow x(-\tau)$

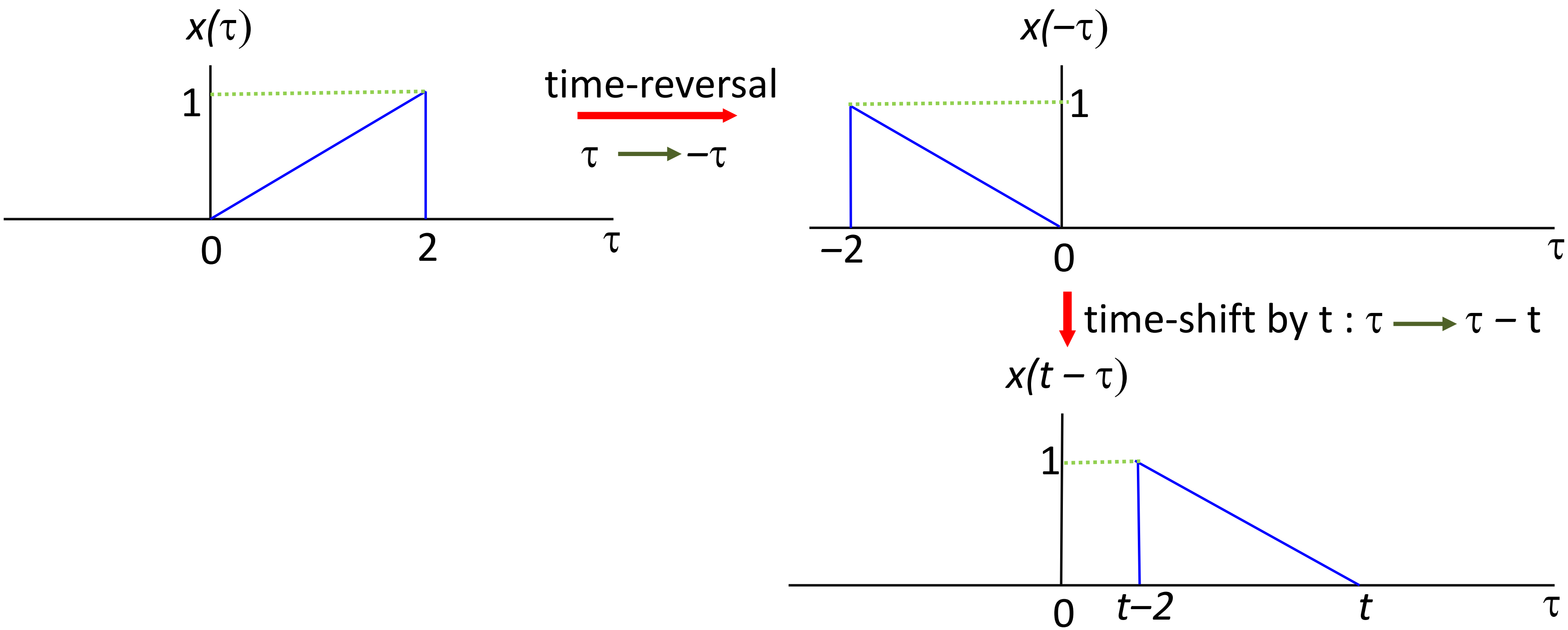
Time-Shift by t : $\tau \longrightarrow \tau - t$

۴) Multiply : $g_t(\tau) = h(\tau)x(t-\tau)$

۵) Integrate : $y(t) = \int_{-\infty}^{\infty} g_t(\tau)d\tau$

- همانند حالت زمان-گسسته، نکتهٔ مهم در روش ترسیمی این است که <u>بازه های مناسبی روی t</u> چنان تعیین کنیم که فرم تابعِ $g_t(\tau)$ را در آن بازه بدانیم و سپس با حدود مناسب انتگرال بگیریم تا به $y(t)$ برسیم.

- **To correctly understand convolution, it is often easier to think graphically.**



- Here, it is assumed that t > 2

Graphical Interpretation of the Convolution Integral

- Convolving two functions involves:
  - flipping or reversing one function in time.
  - sliding this reversed or flipped function over the other.
  - integrating between the times when BOTH functions overlap.

Note the following Examples

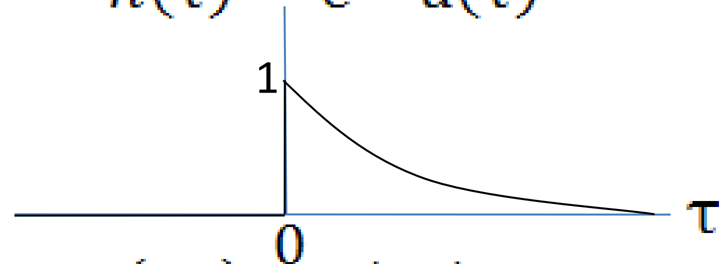
Example 1

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

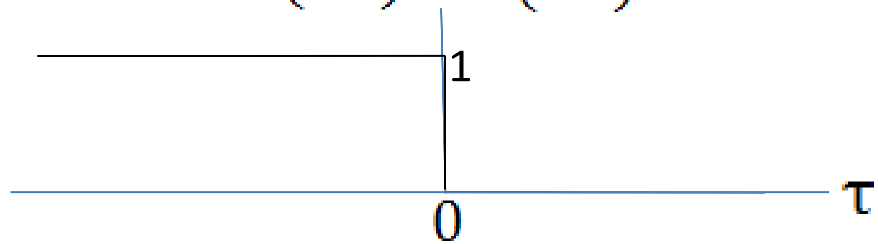
$x(t) = u(t)$ &

$h(t) = e^{-t}u(t)$

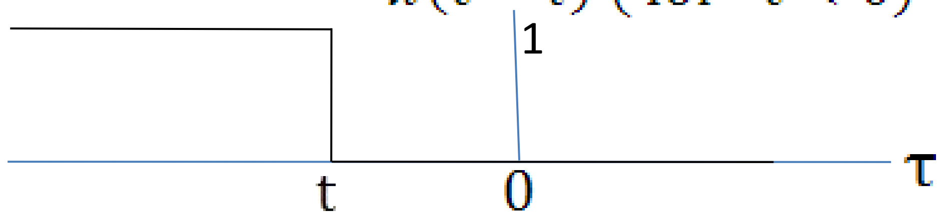
$h(\tau) = e^{-\tau}u(\tau)$



$x(-\tau) = u(-\tau)$



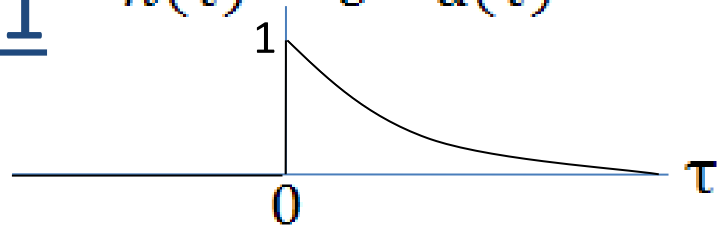
$x(t-\tau)$ (for $t < 0$)



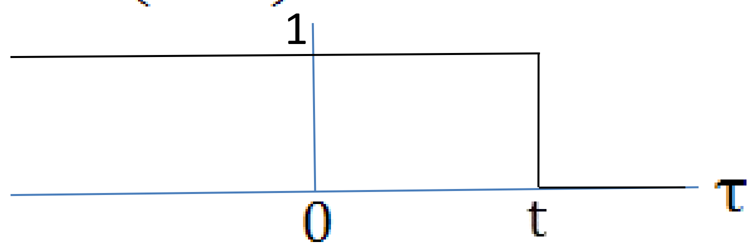
for $t \le 0$: $\quad g_t(\tau) = h(\tau)x(t-\tau) = 0 \quad \forall\tau \;\Rightarrow\; y(t) = 0$

Example 1

$$h(\tau) = e^{-\tau}u(\tau)$$

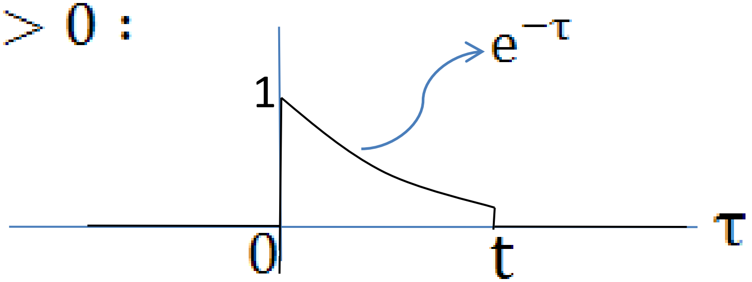


$$x(t-\tau) \text{ for } t > 0$$



$$g_t(\tau)$$

$$t > 0 :$$

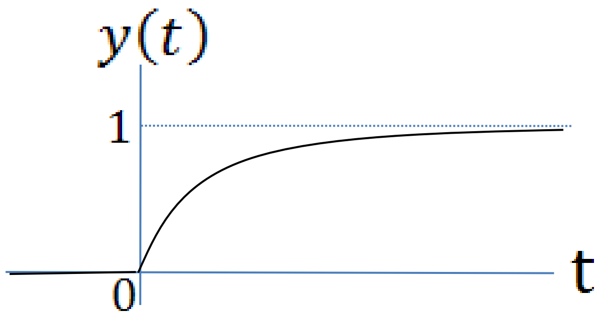


$$y(t) = \int_0^t e^{-\tau}d\tau = -e^{-\tau}\Big|_{\tau=0}^{t} = 1 - e^{-t}$$

Example 1

$$x(t) = u(t) \quad \& \quad h(t) = e^{-t}u(t)$$

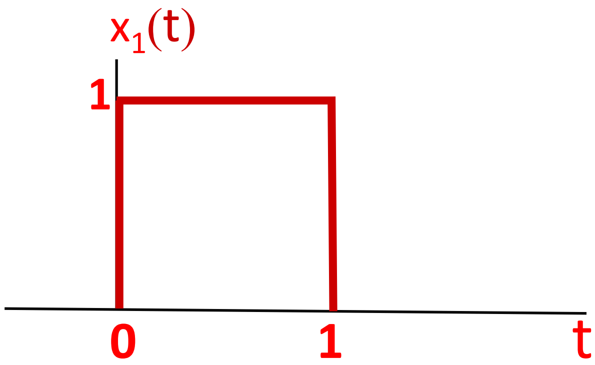
$$y(t) = (1 - e^{-t})u(t)$$

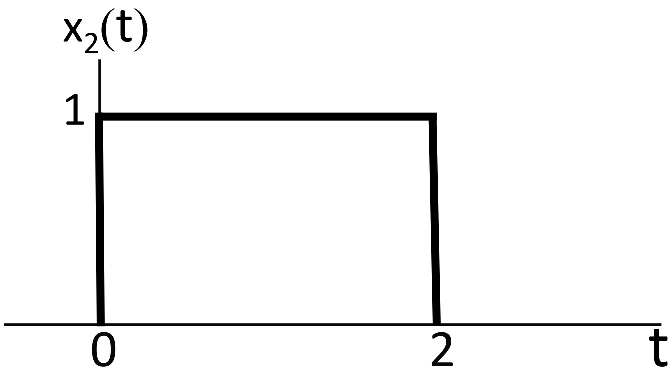


❖ What is this LTI system in EE ?

Example 2

- Convolution of two gate pulses each of height 1



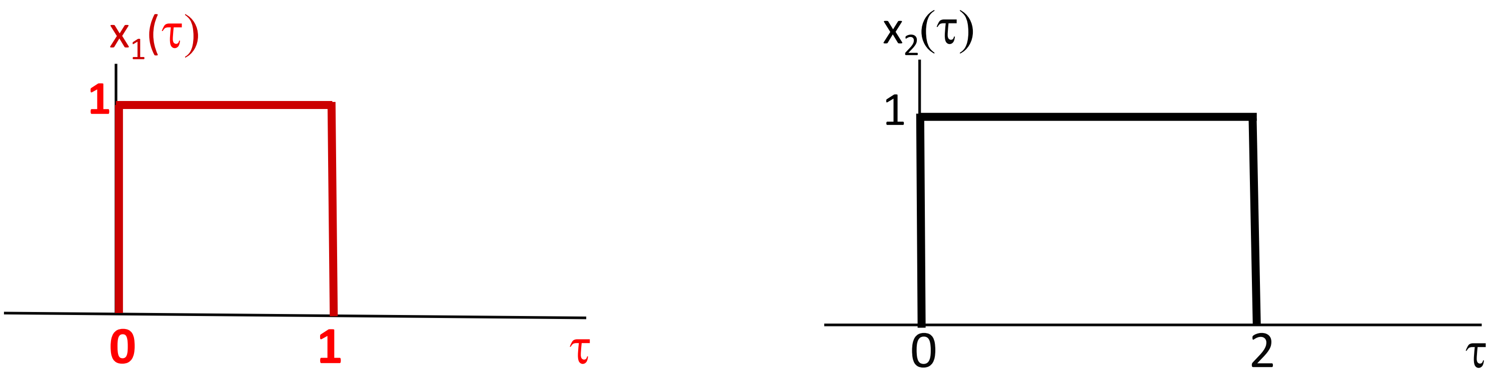


$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

Example 2

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)d\tau$$

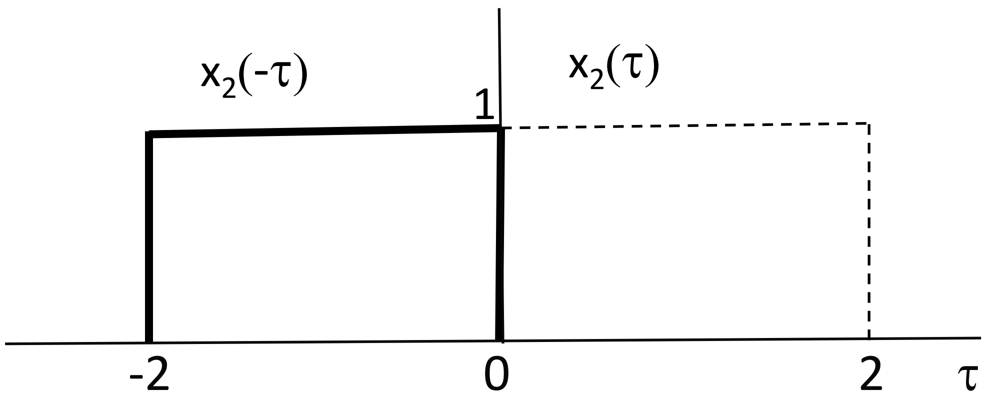
- Name change: t ⟶ τ



Example 2

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$$

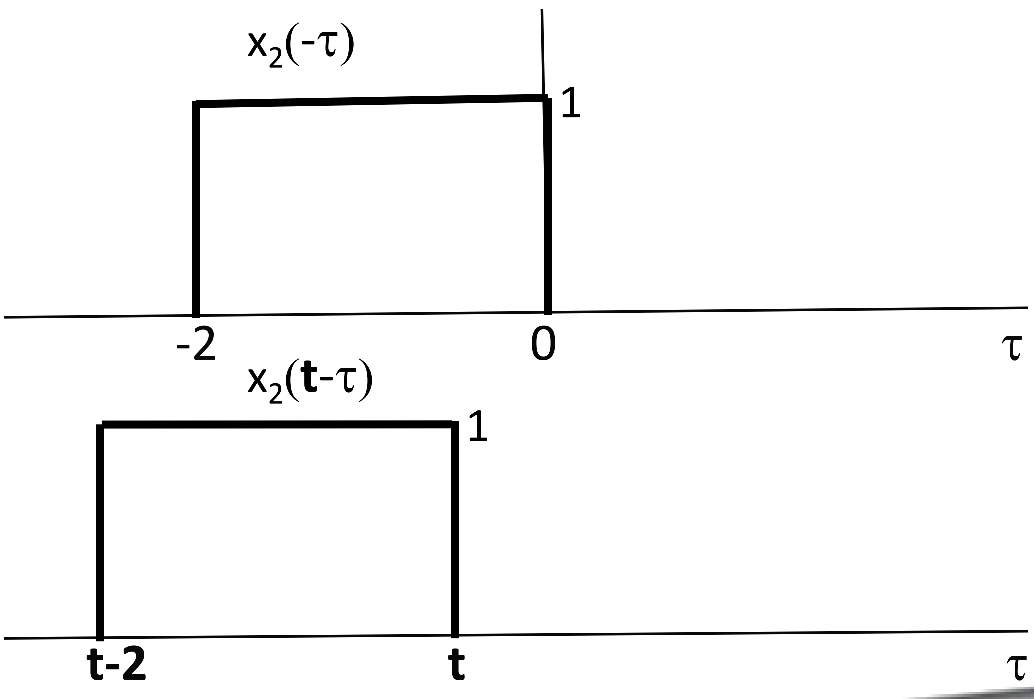
❖ Time reversal: τ ⟶ -τ



Example 2

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$$

❖ Time shift by t: $\tau \longrightarrow \tau - \mathbf{t}$ in $x_2(-\tau)$



Example 2

$$y(t)=x_1(t)*x_2(t)=\int_{-\infty}^{\infty}x_1(\tau)x_2(t-\tau)d\tau$$

❖ Slide $x_2(-\tau)$ over $x_1(\tau)$, Multiply, & Evaluate integral

$$g_t(\tau)\triangleq x_1(\tau)x_2(t-\tau)$$

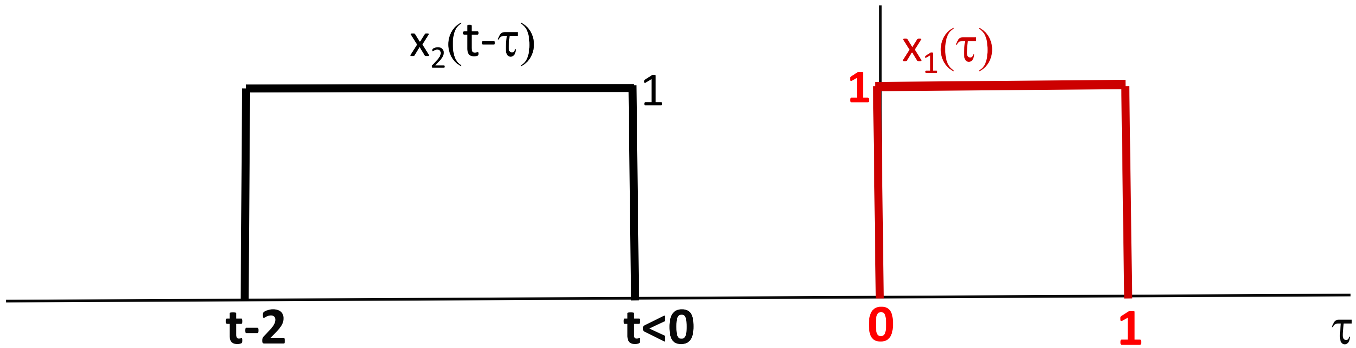
$$y(t)=\int_{-\infty}^{\infty}g_t(\tau)d\tau$$

❑ نکتهٔ مهم در روش ترسیمی: باید بازه های مناسبی روی t چنان تعیین کنیم که فرم تابعی $g_t(\tau)$ را در آن بازه بدانیم و سپس با حدود مناسب انتگرال بگیریم تا به $y(t)$ برسیم.

Example 2

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$$

• for $t < 0$:



$$\Rightarrow g_t(\tau) \triangleq x_1(\tau) x_2(t-\tau) = 0, \quad \forall \tau$$

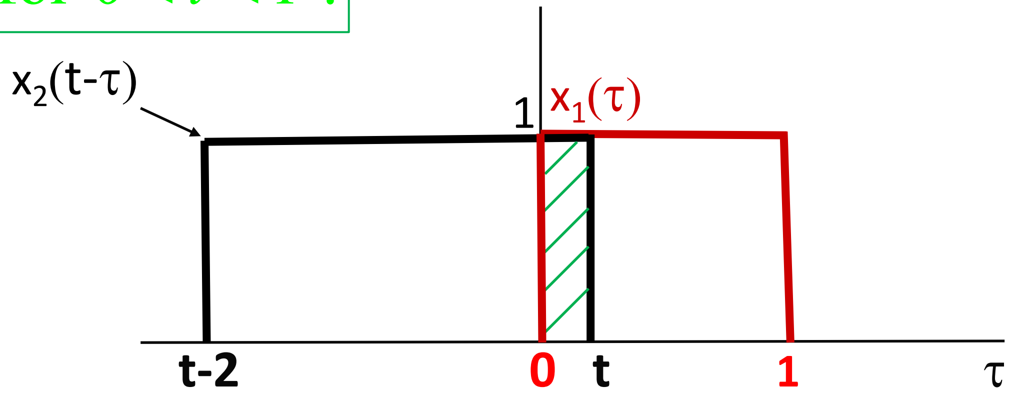
1

$$y(t) = x_1 * x_2 = 0$$

Example 2

$$y(t)=x_1(t)*x_2(t)=\int_{-\infty}^{\infty}x_1(\tau)x_2(t-\tau)d\tau$$

- for $0<t<1$:



$$y(t)=x_1*x_2=\int_0^t 1d\tau=t$$

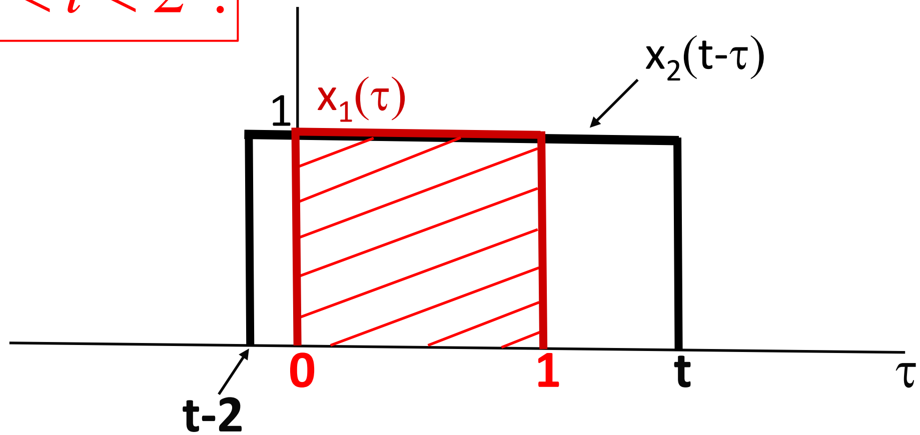
(Area of overlap is increasing linearly)

2

Example 2

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$$

- for $1 < t < 2$:



$$y(t) = x_1 * x_2 = \int_0^1 (1 \times 1) d\tau = \tau \Big|_0^1 = 1$$

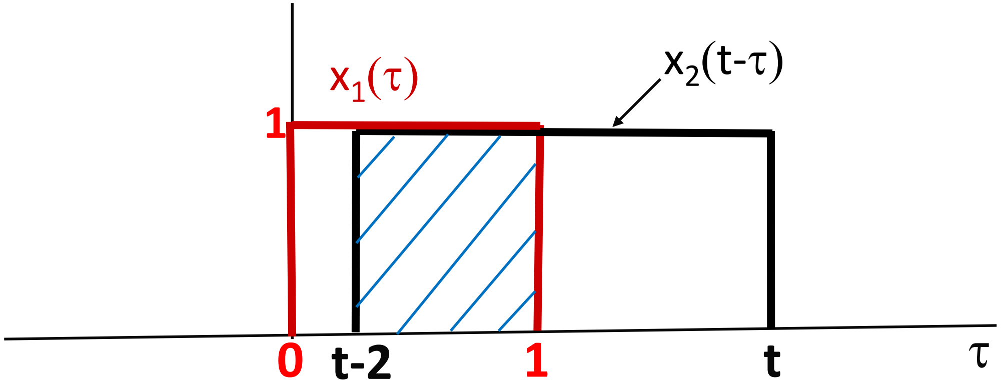
3

(Area of overlap = constant = area of the smaller pulse)

Example 2

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$$

- for $2 < t < 3$:



$$y(t) = x_1 * x_2 = \int_{t-2}^{1} (1\times 1) d\tau = \tau \Big|_{t-2}^{1} = 1-(t-2) = 3-t$$

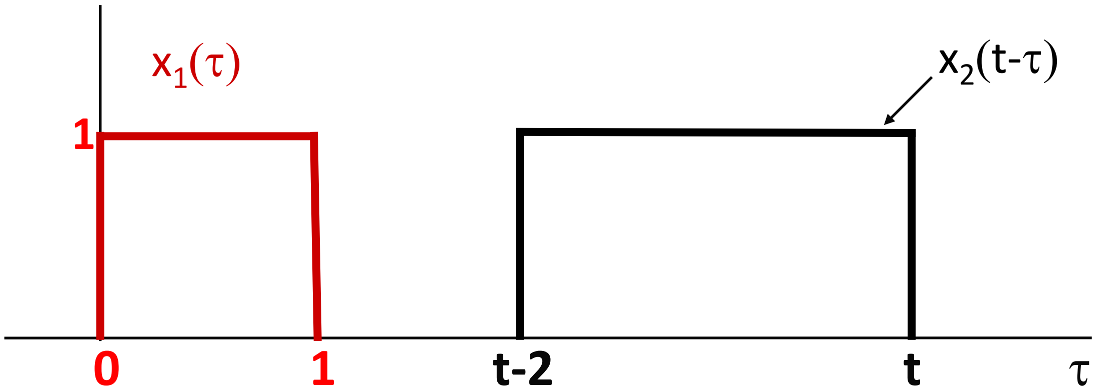
(Area declining linearly: width of shaded area = *1-(t-2)=3-t*)

4

Example 2

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$$

- for $t > 3$:



$$y(t) = x_1 * x_2 = 0$$

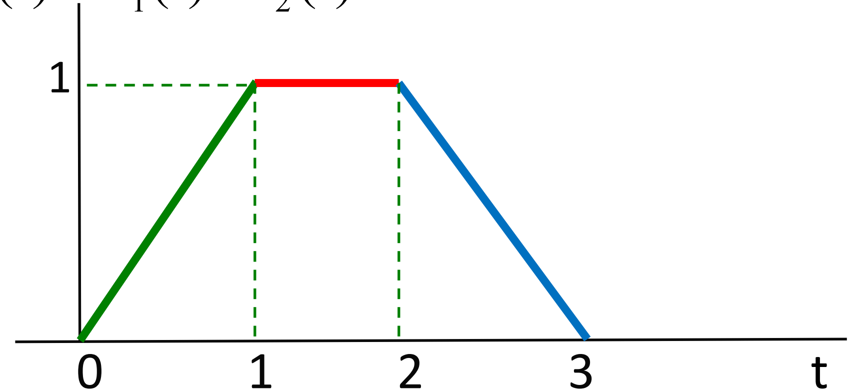
5

(After time *t=3* the convolution integral is zero)

Example 2

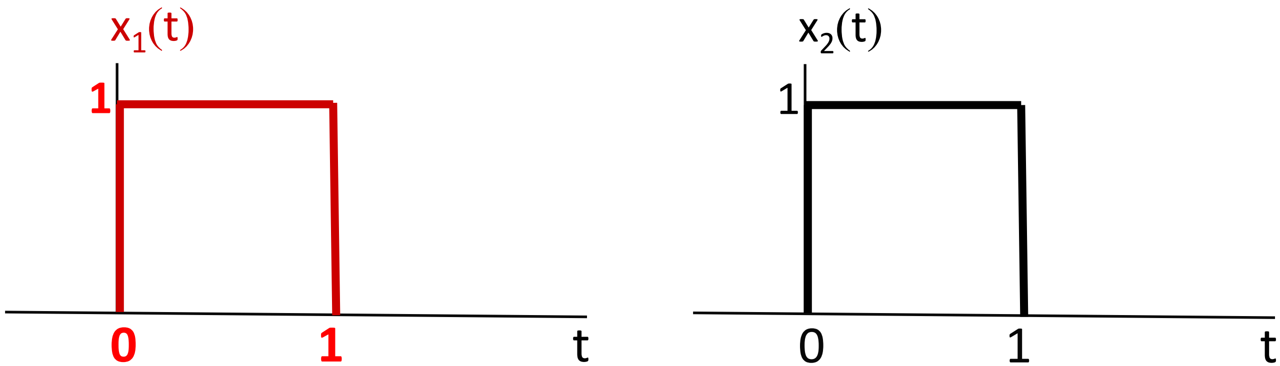
$$y(t)=\begin{cases}0, & t\le 0\\ t, & 0\le t\le 1\\ 1, & 1\le t\le 2\\ 3-t, & 2\le t\le 3\\ 0, & t\ge 3\end{cases}$$

$$y(t)=x_1(t)*x_2(t)$$



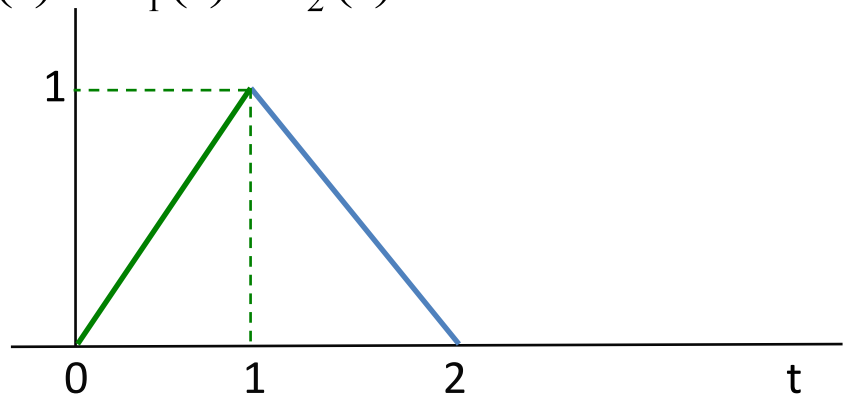
Example 3

- Convolve the following functions



Example 3

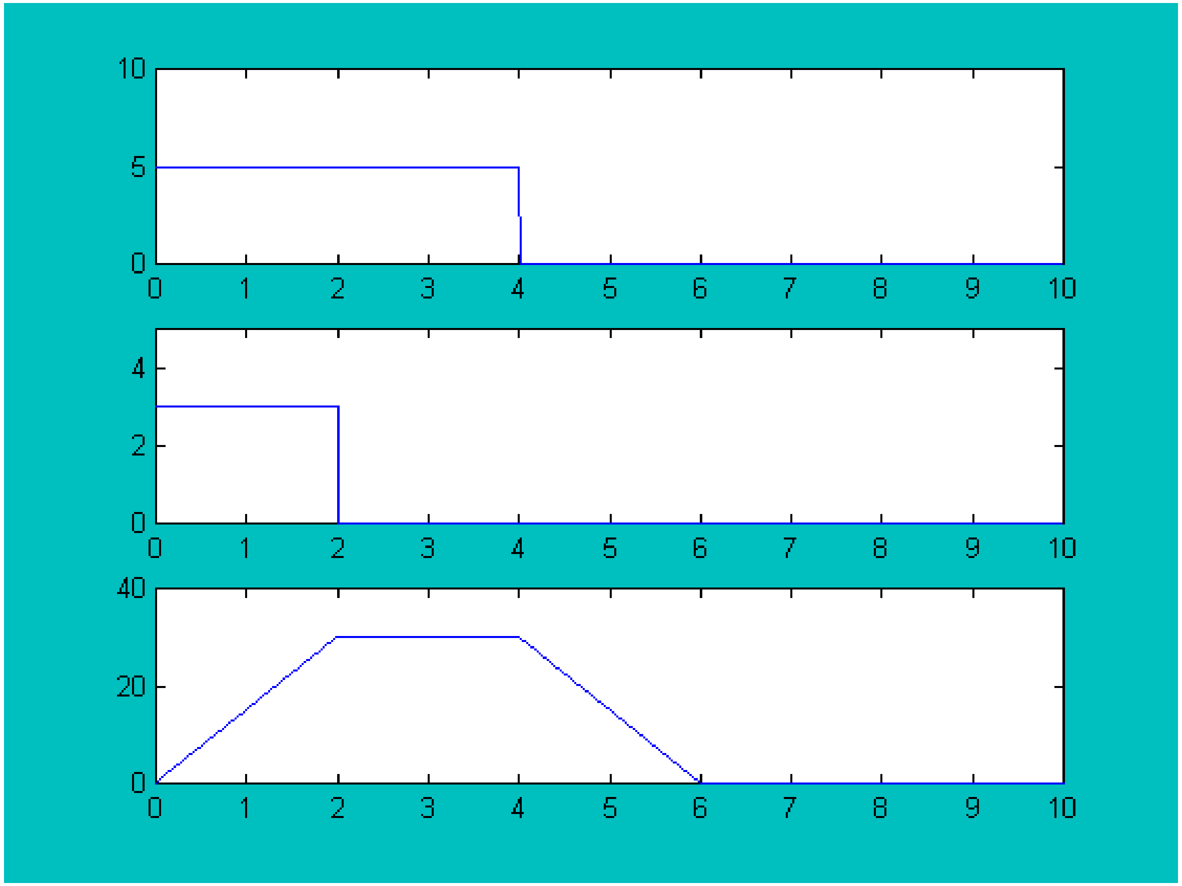
$$y(t) = x_1(t) * x_2(t)$$



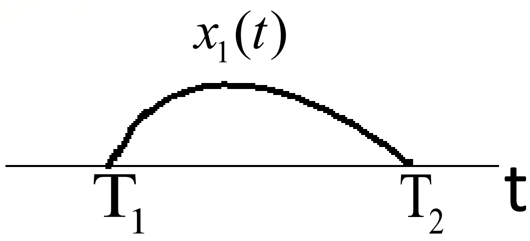
Using MATLAB
for the convolution of two gate pulses

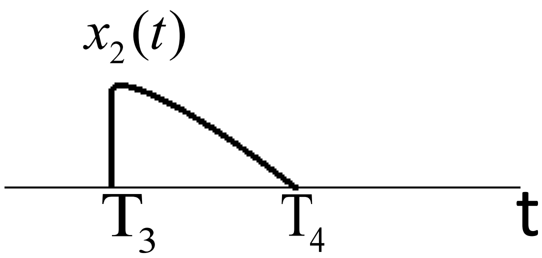
```
tint=0;
tfinal=10;
tstep=.01;
t=tint:tstep:tfinal;
x=5*((t>=0)&(t<=4));
subplot(3,1,1), plot(t,x)
axis([0 10 0 10])
h=3*((t>=0)&(t<=2));
subplot(3,1,2),plot(t,h)
axis([0 10 0 5])
t2=2*tint:tstep:2*tfinal;
y=conv(x,h)*tstep;
subplot(3,1,3),plot(t2,y)
axis([0 10 0 40])
```

Using MATLAB
for the convolution of two gate pulses

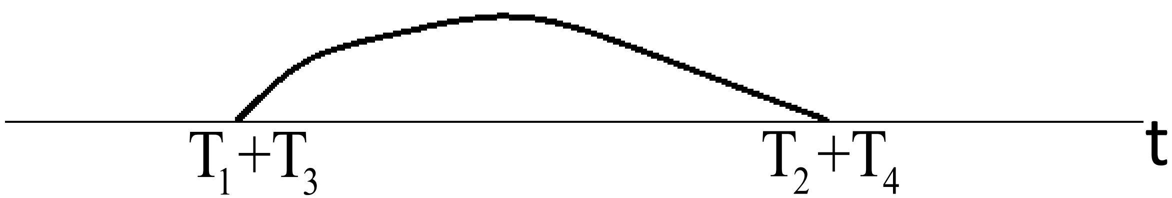


Useful Note:





$$y(t) = x_1(t) * x_2(t)$$



Example 4

Suppose the impulse response of an LTI system is

$$h(t) = 5e^{-2t}u(t).$$

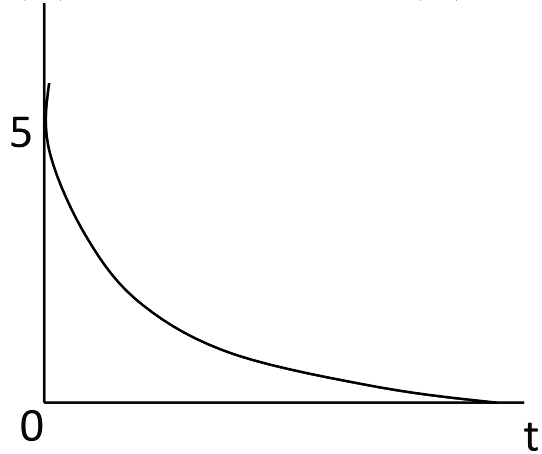
By graphical evaluation of the convolution integral, compute & sketch the output of this system due to an input which is a 4 second pulse of height 3.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

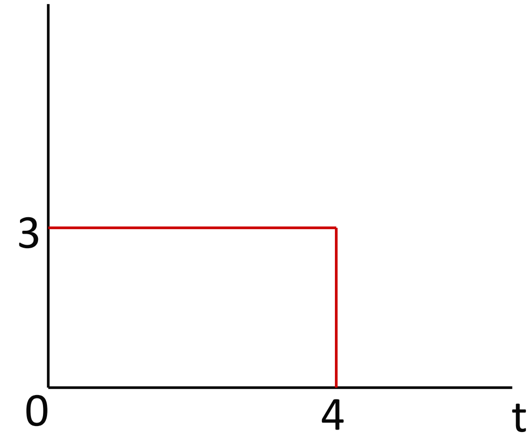
Example 4

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$h(t) = 5e^{-2t}u(t)$$



$$x(t)$$



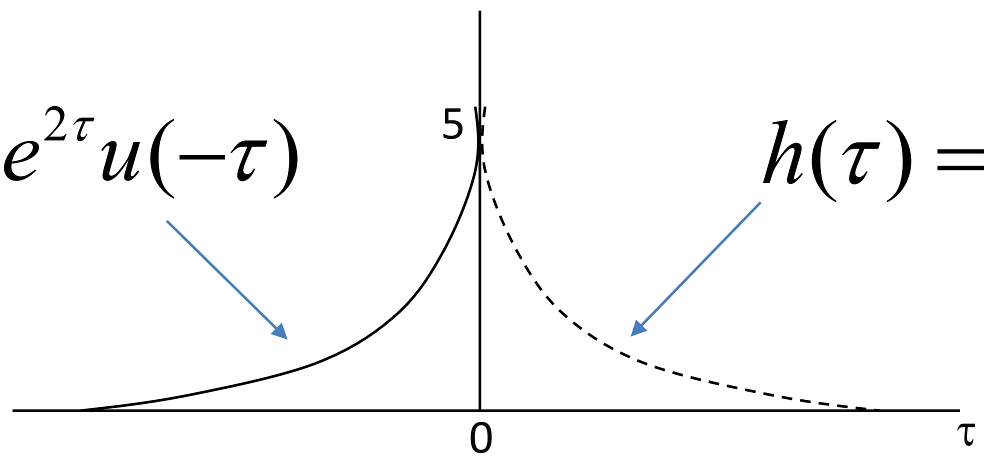
Example 4

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

Reverse $h(\tau)$:

$$h(-\tau) = 5e^{2\tau}u(-\tau)$$

$$h(\tau) = 5e^{-2\tau}u(\tau)$$



Example 4

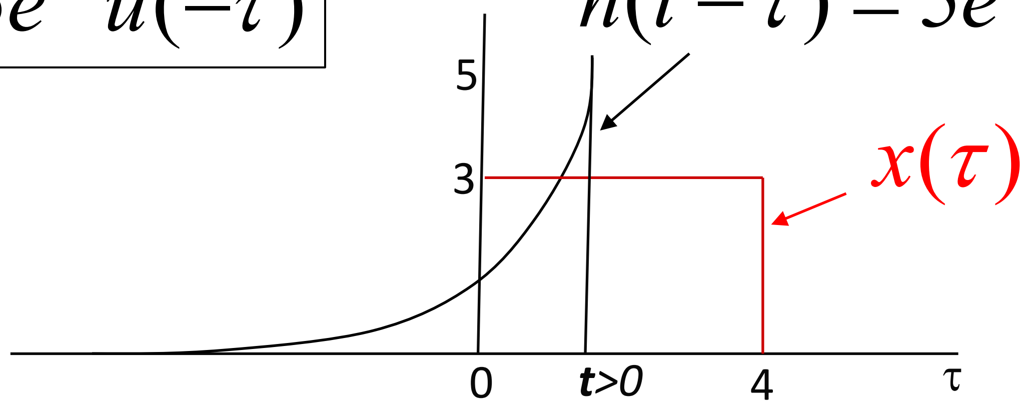
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

Shift the reversed $h(\tau)$ *by t*:

$$\tau \rightarrow \tau - t \quad in \; h(-\tau)$$

$$h(-\tau) = 5e^{2\tau}u(-\tau)$$

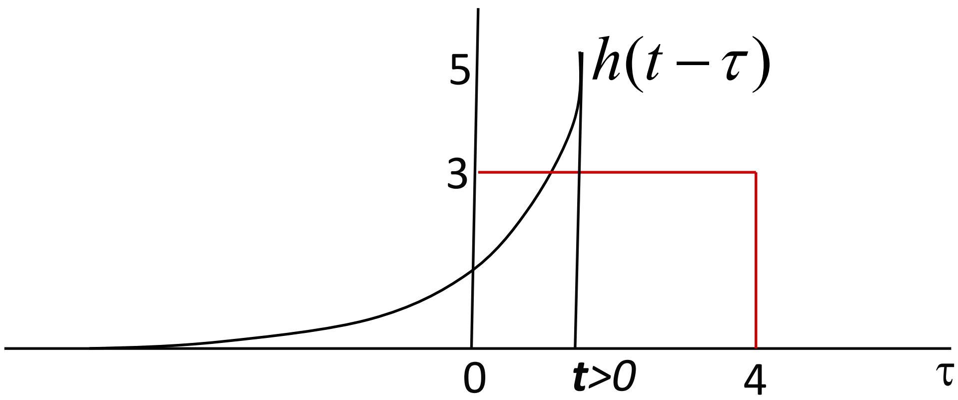
$$h(t-\tau) = 5e^{2(\tau-t)}u(t-\tau)$$



Example 4

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

Performing the integral of the product for **$0 < t < 4$**:



$$h(t-\tau) = 5e^{2(\tau-t)}u(t-\tau)$$

$$\text{output } y(t) = \int_0^t 3 \times 5e^{2(\tau-t)}\,d\tau$$

Example 4

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

So, for **$0 < t < 4$**:

$$y(t) = \int_0^t 15e^{2(\tau - t)} d\tau = 15e^{-2t} \int_0^t e^{2\tau} d\tau$$

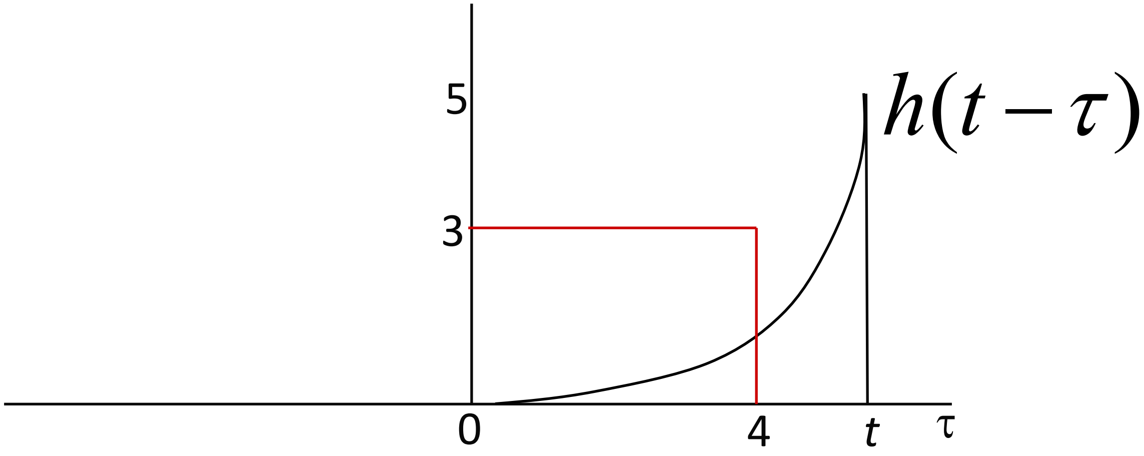
$$= 15e^{-2t} \left[\frac{1}{2} e^{2\tau} \right]_0^t$$

$$\rightarrow y(t) = 7.5\left(1 - e^{-2t}\right)$$

Example 4

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

Performing the integral of the product for **t > 4** :



$$h(t-\tau) = 5e^{2(\tau-t)}u(t-\tau)$$

$$\text{output } y(t) = \int_0^4 3 \times 5e^{2(\tau-t)}\,d\tau$$

Example 4

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

So, for **t > 4** :

$$y(t) = \int_0^4 15e^{2(\tau-t)}d\tau$$

$$= 15e^{-2t}\int_0^4 e^{2\tau}d\tau$$

$$= 15e^{-2t}\left[\frac{1}{2}e^{2\tau}\right]_0^4$$

$$= 7.5e^{-2t}\left(e^8 - 1\right)$$

$$= 7.5\left(1 - e^{-8}\right)e^{-2(t-4)}$$

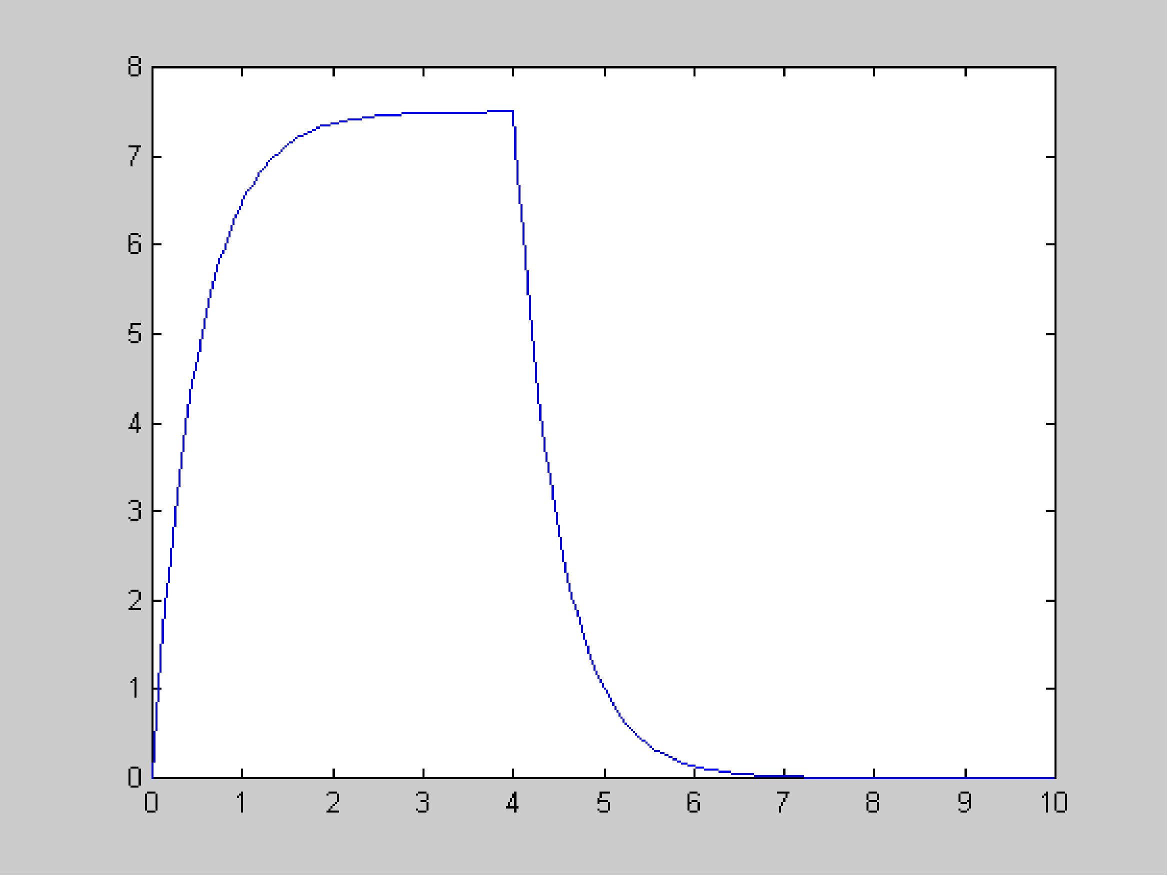
Example 4

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$y(t) = \begin{cases} 0, & t \le 0 \\ 7.5\left(1 - e^{-2t}\right), & 0 \le t \le 4 \\ 7.5\left(1 - e^{-8}\right)e^{-2(t-4)}, & t \ge 4 \end{cases}$$

Example 4

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$



- Convolution is commutative.
- So, the actions of flipping and shifting can be applied to EITHER function:

$$x(t)*h(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau=$$

$$h(t)*x(t)=\int_{-\infty}^{+\infty}h(\tau)x(t-\tau)d\tau$$

Example 5

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

- Repeat example 4 by flipping and shifting *x(t)* rather than *h(t)*.

for $0 < t < 4$:

$$y(t) = \int_0^t 5e^{-2\tau} \times 3d\tau$$

$$= \int_0^t 15e^{-2\tau} d\tau = \left[-7.5e^{-2\tau}\right]_0^t$$

$$= 7.5\left(1 - e^{-2t}\right)$$

Example 5

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

for $t > 4$:

$$y(t) = \int_{t-4}^{t} 15e^{-2\tau}\,d\tau = 15\left[\frac{-1}{2}e^{-2\tau}\right]_{t-4}^{t}$$

$$\rightarrow y(t) = 7.5\left(e^{-2(t-4)} - e^{-2t}\right) = 7.5\left(1 - e^{-8}\right)e^{-2(t-4)}$$

Example 5

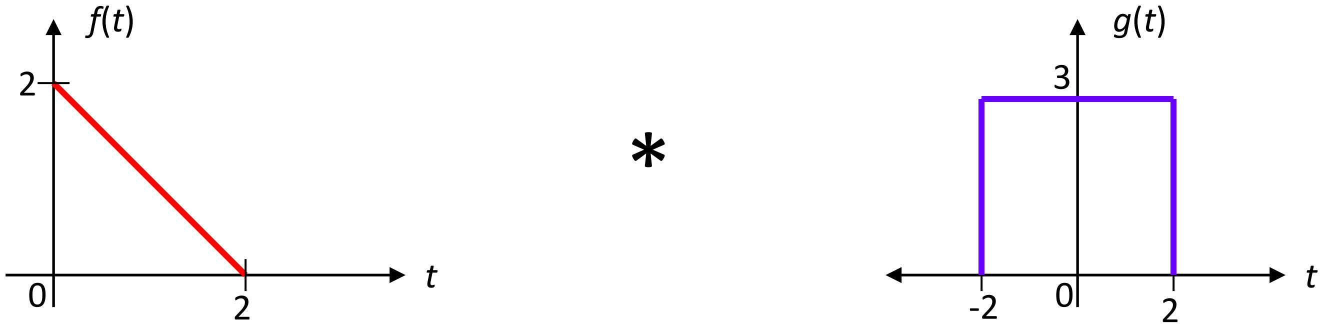
$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

$$y(t) = \begin{cases} 0, & t \le 0 \\ 7.5\left(1-e^{-2t}\right), & 0 \le t \le 4 \\ 7.5\left(1-e^{-8}\right)e^{-2(t-4)}, & t \ge 4 \end{cases}$$

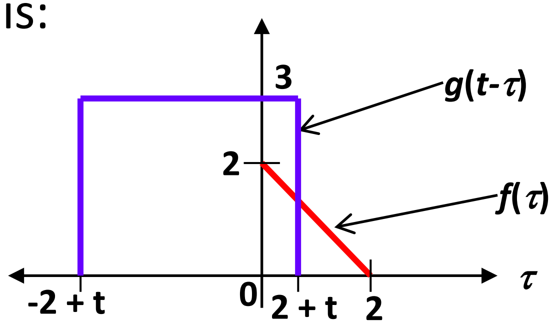
Same result as before!

Example 6

- Convolve the following two functions:



- Replace t with τ in $f(t)$ and $g(t)$
- Choose to flip and slide $g(\tau)$ since it is simpler and symmetric
- Functions overlap like this:



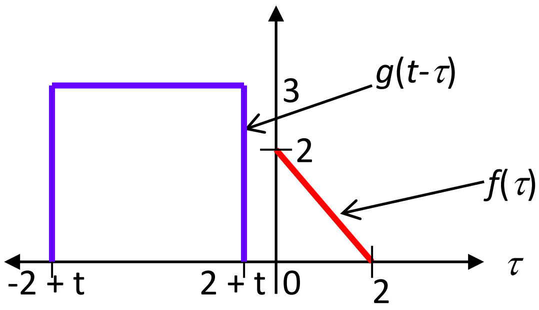
Example 6

This convolution can be divided into 5 parts

1

$t < -2$

- Two functions do not overlap
- Area under the product of the functions is zero.

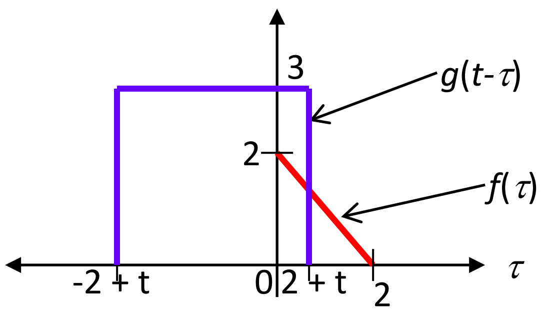


Example 6

2

$-2 \leq t < 0$

- Part of *g* overlaps part of *f*
- Area under the product of the functions is:



$$\int_0^{2+t} 3(-\tau+2)d\tau = 3\left(-\frac{\tau^2}{2}+2\tau\right)\Bigg|_0^{2+t} = -\frac{3(2+t)^2}{2}+6(2+t) = -\frac{3t^2}{2}+6$$

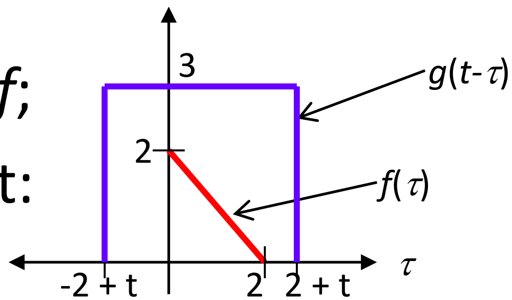
Example 6

3

$0 \leq t < 2$

- Here, *g* completely overlaps *f*;

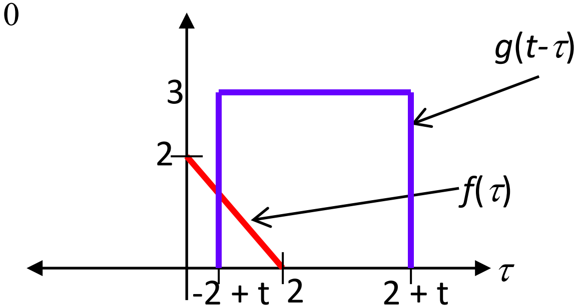
area under the product is just:



$$\int_0^2 3(-\tau+2)d\tau = 3\left(-\frac{\tau^2}{2}+2\tau\right)\Bigg|_0^2 = 6$$

4

$2 \leq t < 4$



- Part of *g* and *f* overlap;

Calculated similarly to $-2 \leq t < 0$

Example 6

5

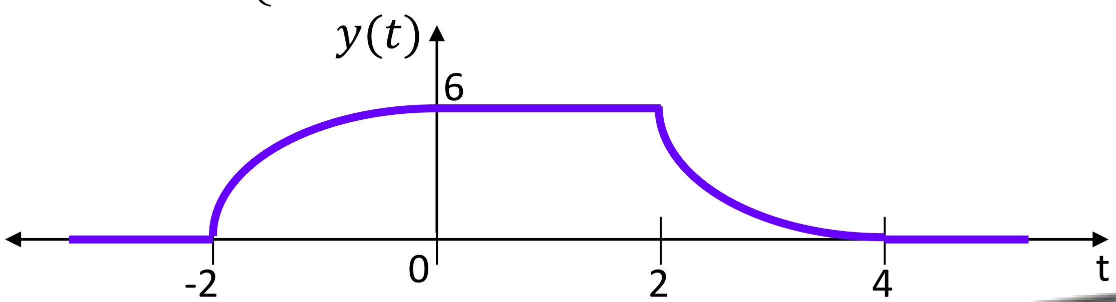
$t \geq 4$

- *g* and *f* **do not overlap**;
  area under their product is zero.

Example 6

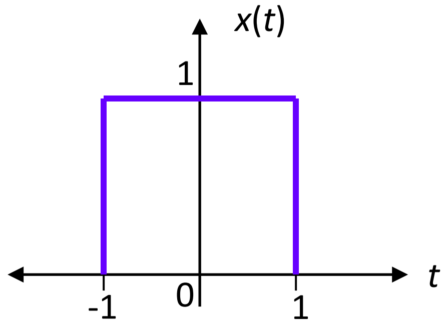
- Result of convolution (5 intervals of interest):

$$y(t) = f(t) * g(t) = \begin{cases} 0, & \text{for } t \le -2 \\ -\frac{3}{2}t^2 + 6, & \text{for } -2 \le t \le 0 \\ 6, & \text{for } 0 \le t \le 2 \\ \frac{3}{2}t^2 - 12t + 24, & \text{for } 2 \le t \le 4 \\ 0, & \text{for } t \ge 4 \end{cases}$$

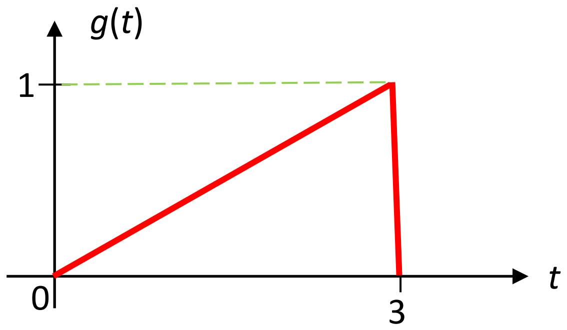


Example 7

- Convolve the following two functions:

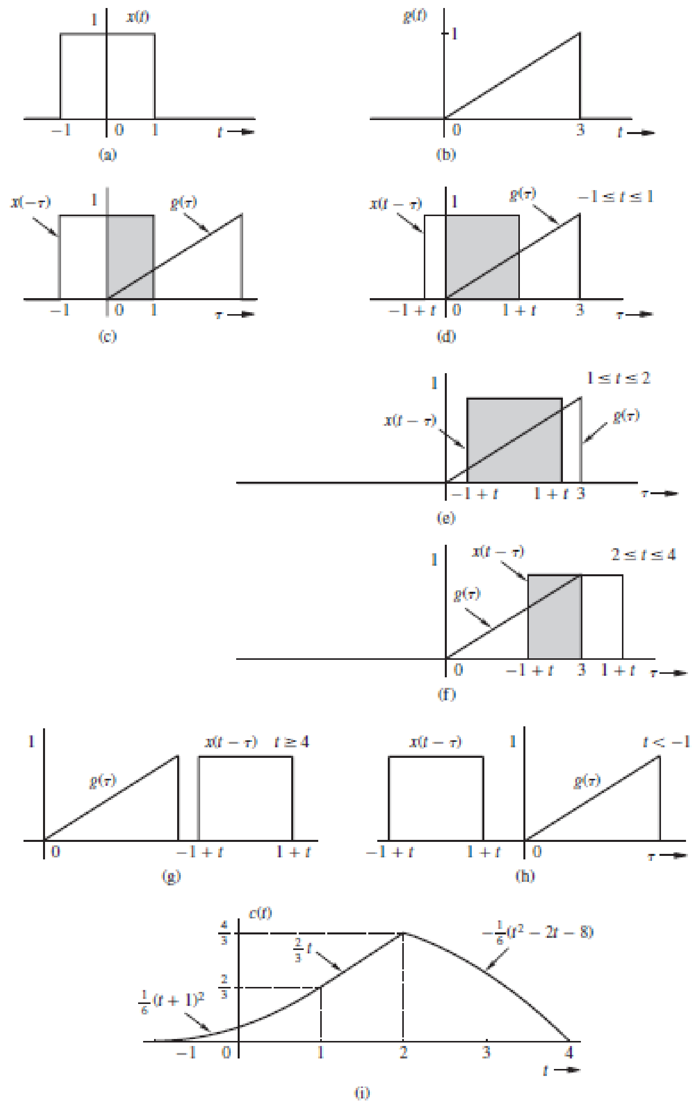


*



- Replace t with τ in $x(t)$ and $g(t)$
- Choose to flip and slide $x(\tau)$ since it is simpler and symmetric.

Example 7


x(t)
(a)
g(t)
(b)
x(−τ)
g(τ)
(c)
x(t − τ)
−1 ≤ t ≤ 1
(d)
1 ≤ t ≤ 2
(e)
2 ≤ t ≤ 4
(f)
t ≥ 4
(g)
t < −1
(h)
c(t)
$\frac{2}{3}t$
$\frac{1}{6}(t+1)^2$
$-\frac{1}{6}(t^2-2t-8)$
(i)

Example 8

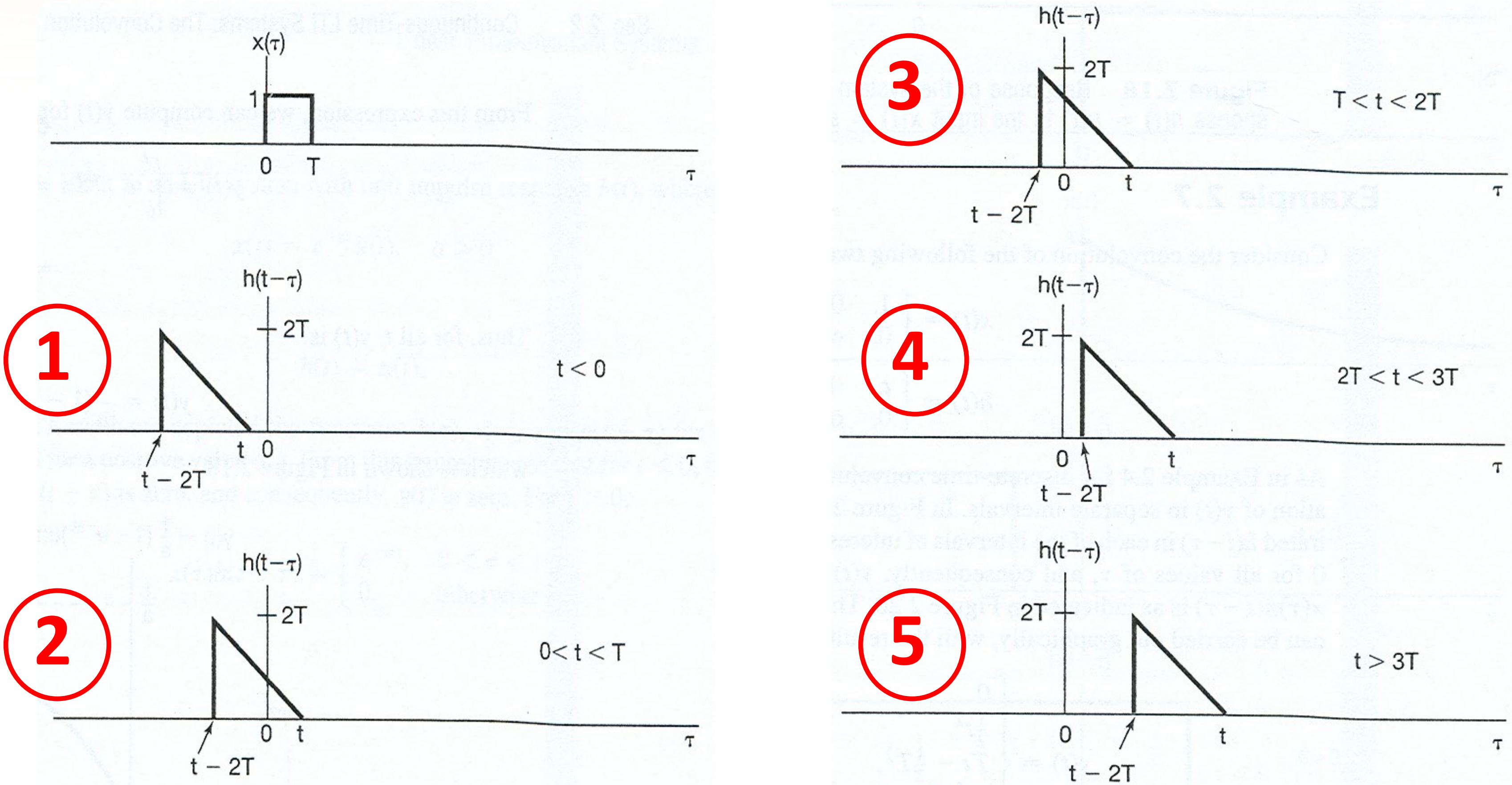
$$x(t)=\begin{cases}1, & 0<t<T\\ 0, & \text{otherwise}\end{cases}$$

$$h(t)=\begin{cases}t, & 0<t<2T\\ 0, & \text{otherwise}\end{cases}$$

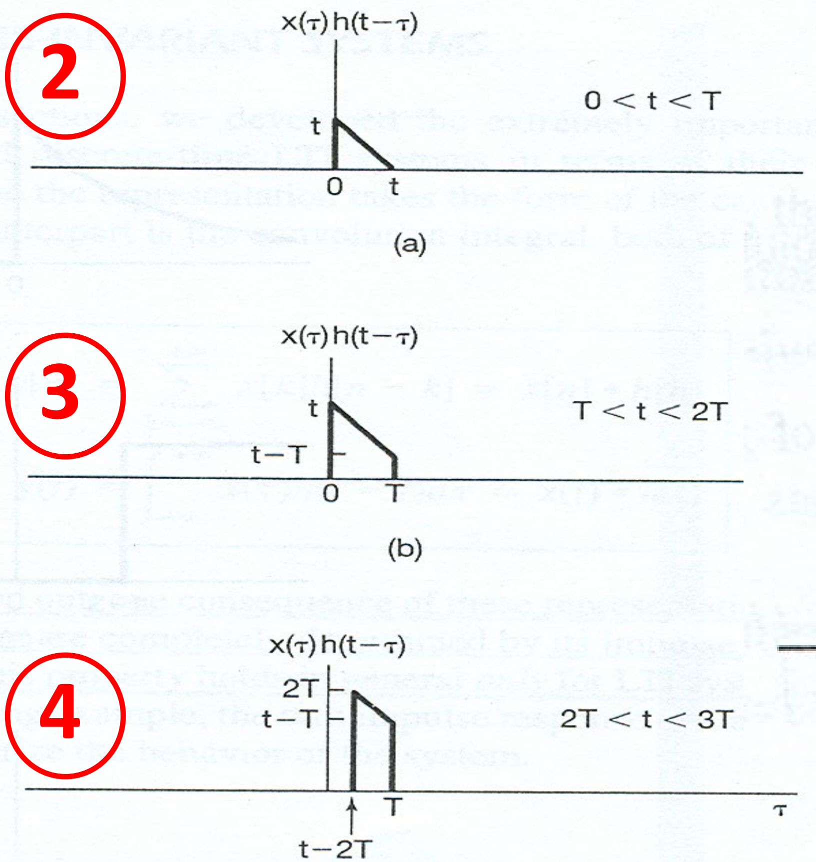
$$\downarrow$$

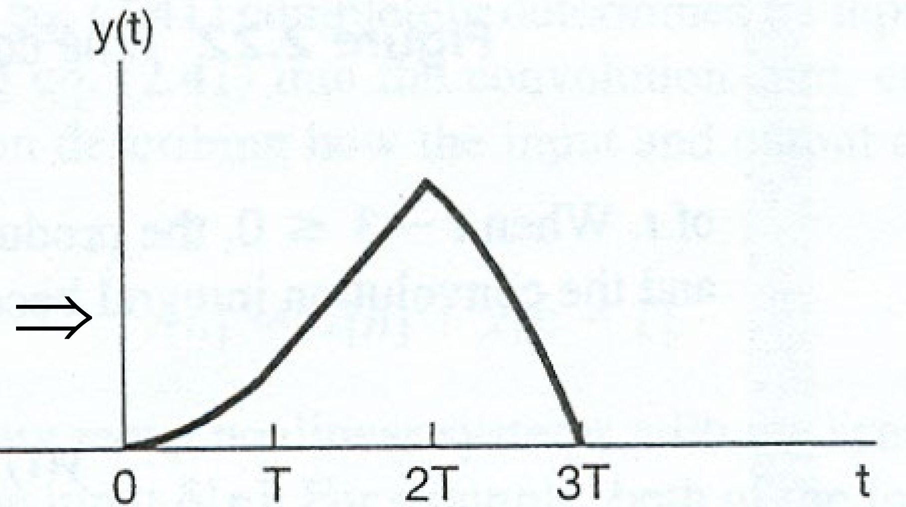
$$y(t)=\begin{cases}0, & t<0\\ \frac{1}{2}t^2, & 0<t<T\\ Tt-\frac{1}{2}T^2, & T<t<2T\\ -\frac{1}{2}t^2+Tt+\frac{3}{2}T^2, & 2T<t<3T\\ 0, & 3T<t\end{cases}$$

Example 8


x(τ)
1
0
T
τ
1
h(t−τ)
2T
t < 0
t
0
t − 2T
τ
2
h(t−τ)
2T
0< t < T
0
t
t − 2T
τ
3
h(t−τ)
2T
T < t < 2T
0
t
t − 2T
τ
4
h(t−τ)
2T
2T < t < 3T
0
t
t − 2T
τ
5
h(t−τ)
2T
t > 3T
0
t
t − 2T
τ

Example 7


2
x(τ)h(t−τ)
0 < t < T
t
0
t
(a)
3
x(τ)h(t−τ)
t
T < t < 2T
t−T
0
T
(b)
4
x(τ)h(t−τ)
2T
t−T
2T < t < 3T
T
τ
t−2T

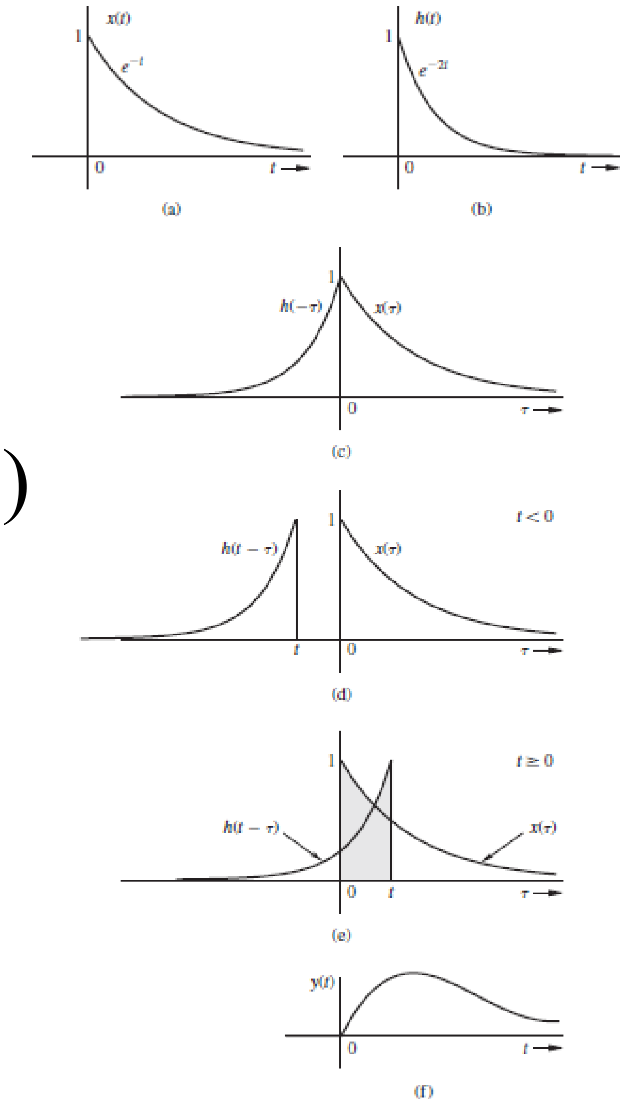

⇒
y(t)
0
T
2T
3T
t

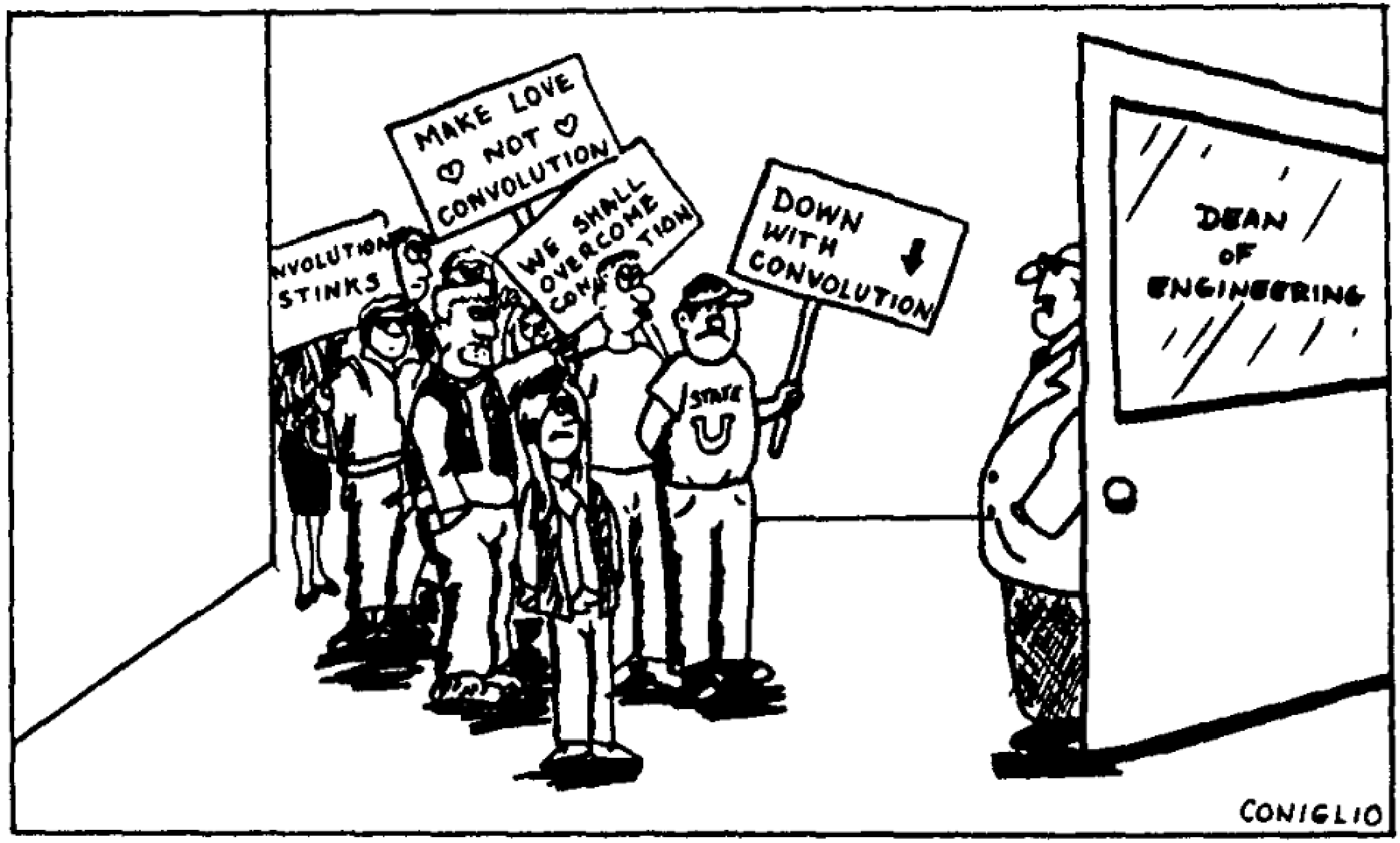
Example 9

$$x(t)=e^{-t}u(t)$$

$$h(t)=e^{-2t}u(t)$$

$$y(t)=(e^{-t}-e^{-2t})u(t)$$





Convolution: its bark is worse than its bite!

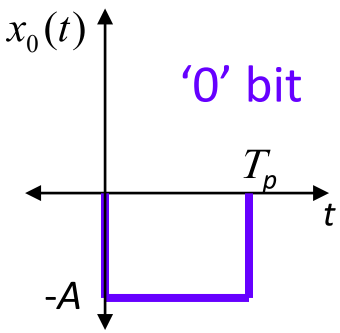
- Illustration of an undesired effect of convolution in **Digital Communication over Band-limited Channels**

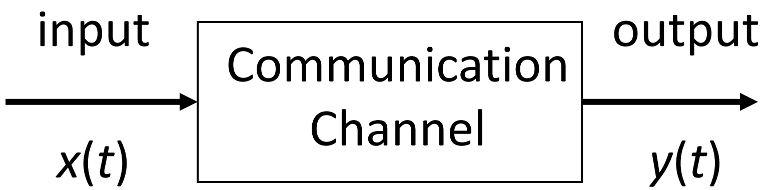
ISI

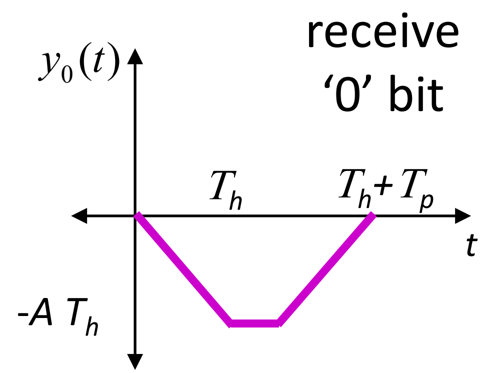
Inter-Symbol Interference

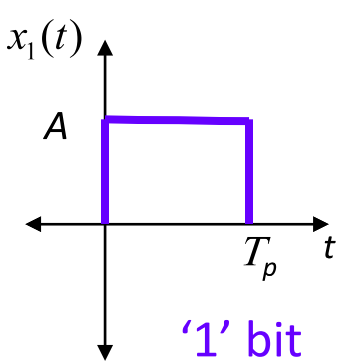
Inter-Symbol Interference

- Transmission over communication channel (e.g. telephone line) is **analog**.

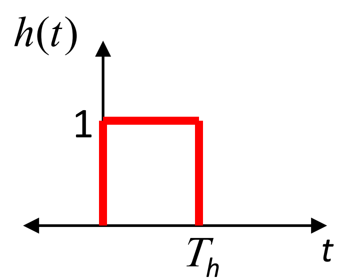


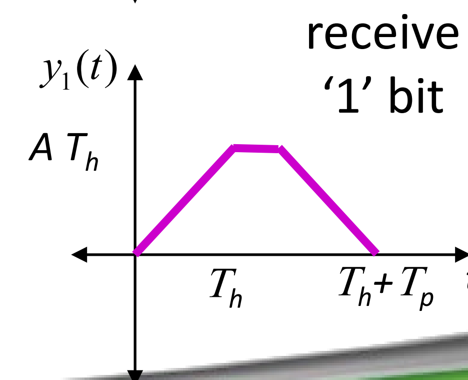






Model channel as LTI system with impulse response h(t)

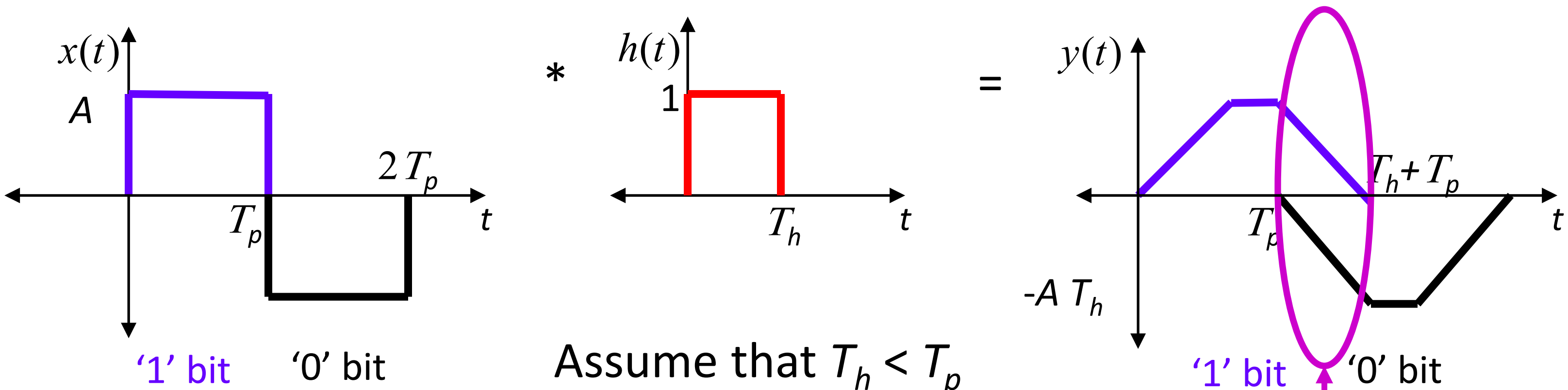




Assume that $T_h < T_p$

Inter-Symbol Interference

- Transmitting two bits (pulses) back-to-back will cause overlap (**interference**) at the receiver



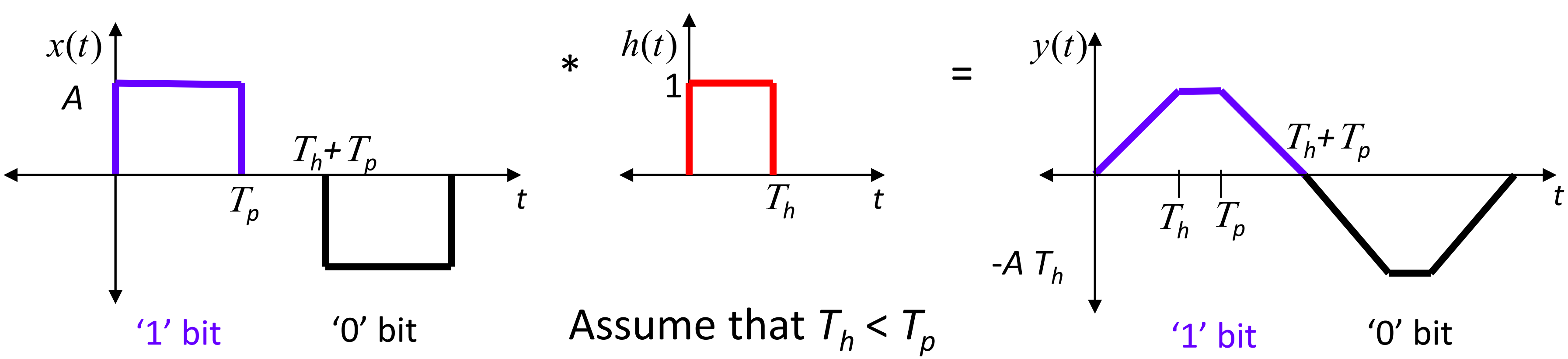
Assume that $T_h < T_p$

Inter-symbol interference

- How do we prevent inter-symbol interference (**ISI**) at the receiver?

Inter-Symbol Interference

- Prevent inter-symbol interference by waiting T_h seconds between pulses (called a guard period).



Assume that $T_h < T_p$

- Disadvantages?...